

# From Cash Flows to Real Options

By

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# Agenda

- From cash flows to real options – finite-lived project
- NPV forces false mutually exclusive alternatives
- Examples of real option cases

## **The Problem of Irregular cash flows**

**1. What does one do when the cash flows of a project or firm follow an irregular stochastic process?**

**2. Answer:**

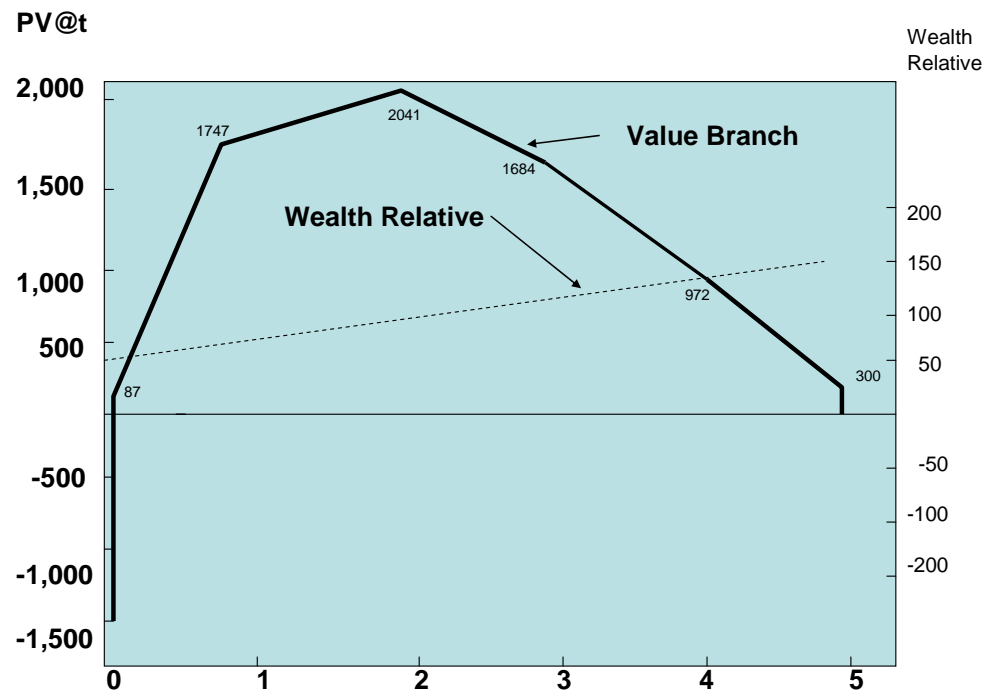
- a. The present value will be a portfolio of futures contracts**
- b. Use the MAD assumption**
- c. Use replicating portfolios and proportional dividends**

**Table 1 -- Expected Free Cash Flows for a 5-Year Project**

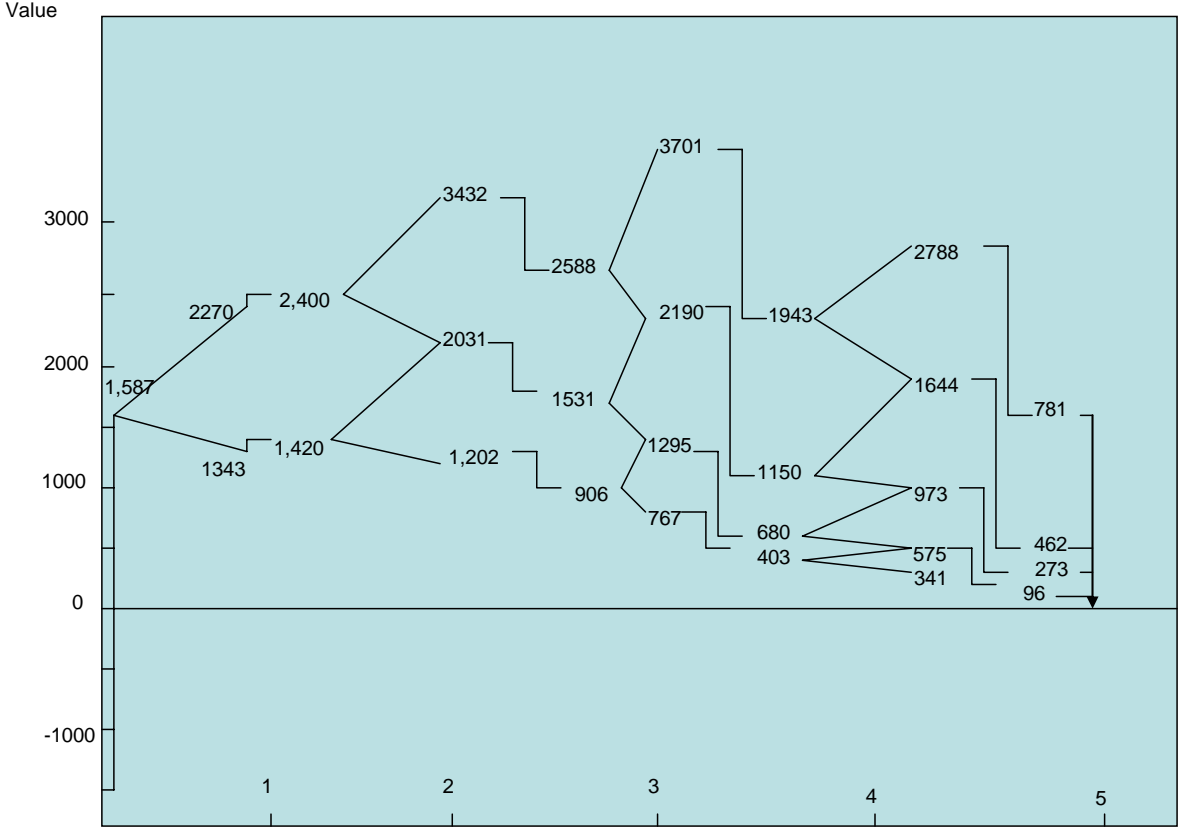
<i>Yr(t)</i>	<i>E(FCF)</i>	<i>PV@10%</i>	<i>PV@Yr.0</i>	<i>PV@Yr.t</i>	<i>PV ex-div.</i>	<i>Wealth Relative</i>	<i>Return</i>
0	-1,500	1.000	-1,500.00	87.30	1,587.30	87.30	
1	-100	.909	-90.90	1,746.53	1,846.53	96.03	10%
2	500	.826	413.00	2,031.18	1,531.18	105.63	10%
3	800	.751	600.80	1,684.30	884.30	116.19	10%
4	700	.683	478.10	972.73	272.73	127.81	10%
5	300	.621	<u>186.30</u>	300.00	0	140.59	10%
			87.30				

Note: Wealth relative in year 2 is -\$1,500 brought forward at the cost of capital for 2 years, plus the year one investment brought forward for 1 year, plus PV@yr. t from column 5, all of which adds up to \$106.

**Figure 2 – Project Value Branch and Wealth Relative**

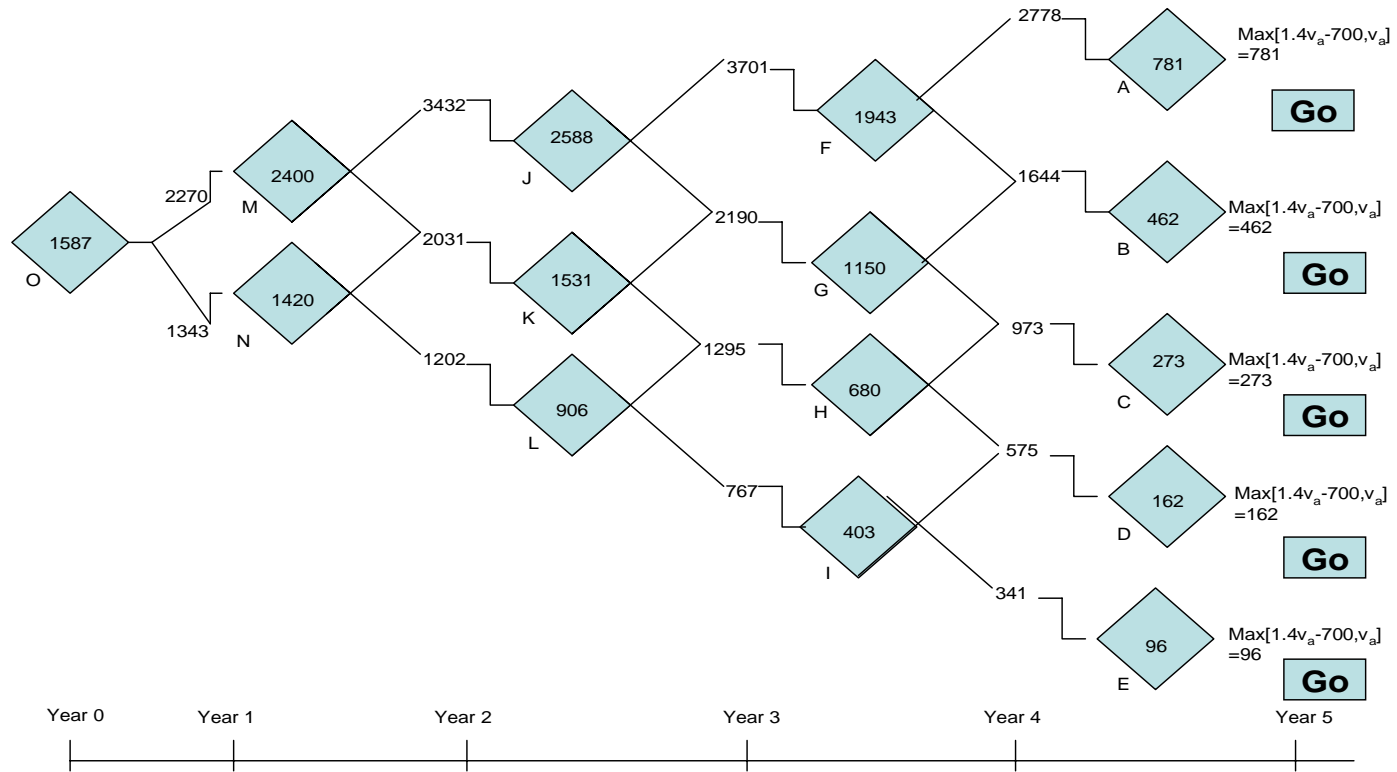


### Figure 3 – Project Value Tree

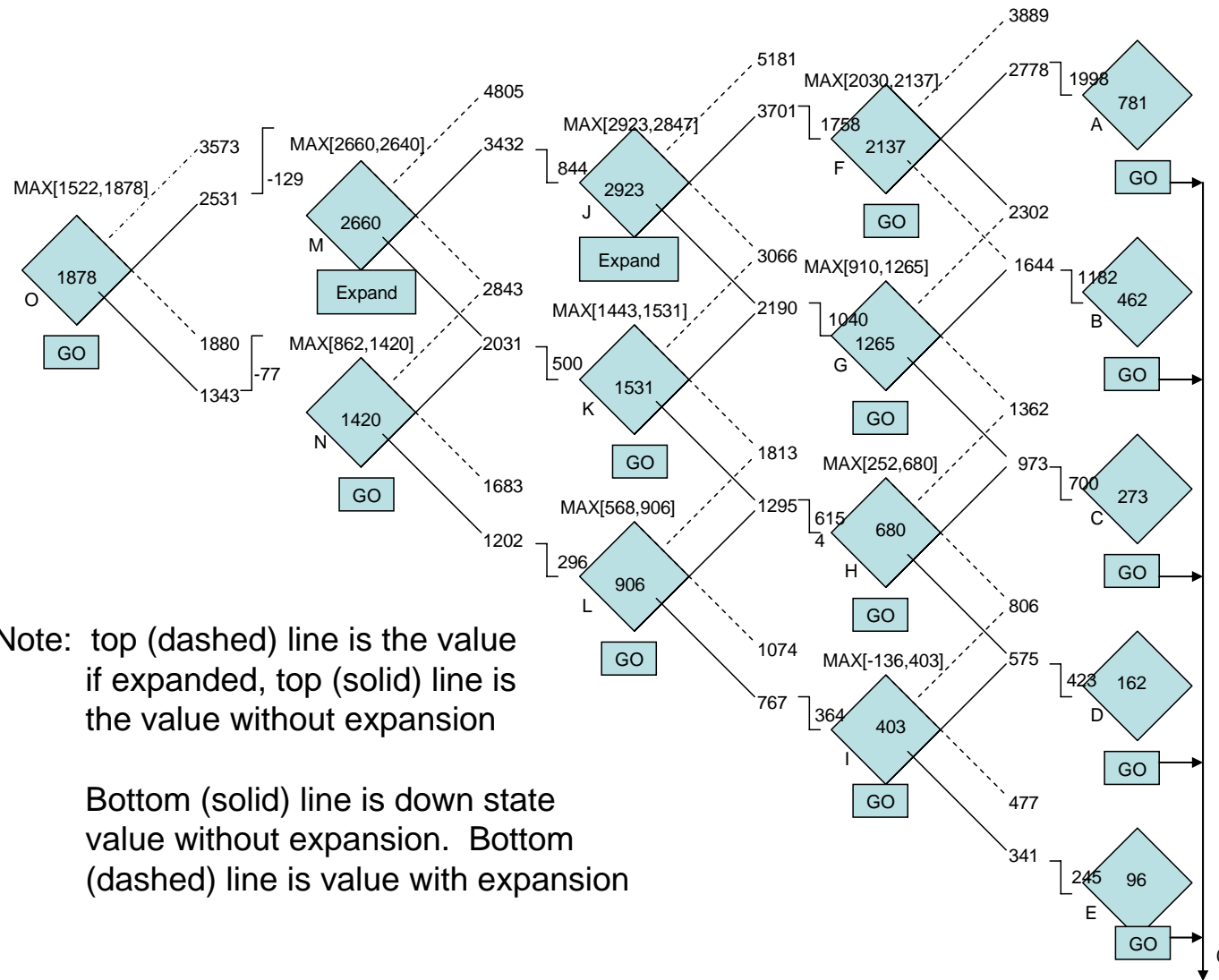


E(Cash Flow)	-1500	-100	500	800	700	300
PV@t	1587	1747	2031	1684	973	300
Percent Payout	NA	-5.7%	24.6%	47.5%	71.9%	100%

# Figure 4 - Value Tree with Option to Expand at End Nodes



# Figure 5 - Real Option Solution



## Takeaways

- Three assumptions
  - MAD
  - Properly anticipated prices
  - No arbitrage
- Link between spreadsheet and binomial tree

# Estimating Volatility

Remember that the volatility is the standard deviation of the underlying project or firm. If you are working on a switching option on a gold mine:

1. You want to estimate the volatility of the mine
  - it is not the same as the volatility of gold or gold futures
  - it is not the same as the volatility of the equity of the company
2. There are 3 approaches
  - Use a Monte Carlo analysis that is spreadsheet based
  - $\ln[V(1)/V(0)] = r$
  - Use a subjective approach
  - Use historical data

# Real Options Analysis

## Cases

1. Development of a coal lease
2. Cancelable operating lease
3. How real options replaced scenarios
4. Phased construction
5. Modularity
6. Peak Load Power

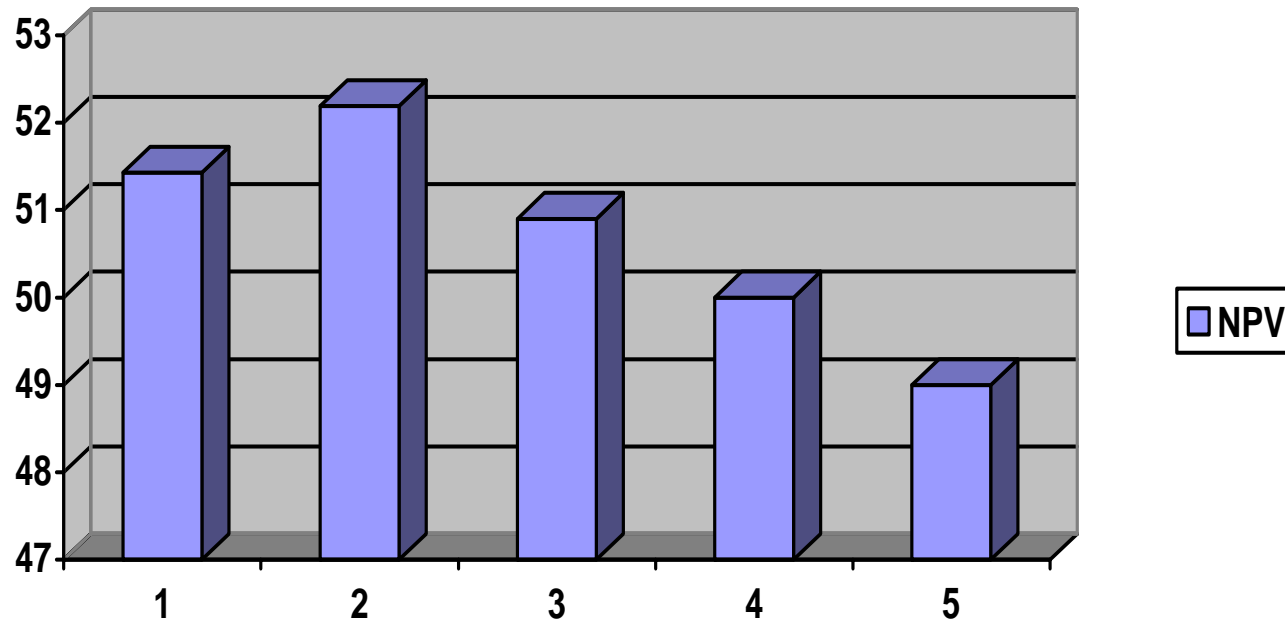
## Case 1: Deferral case facts

1. Bidding on monopoly rights to develop a coal lease property
2. 5 years to develop
3. Near-the-money option
4. NPV analysis of 5 mutually exclusive alternatives suggests that developing after 2 years has the highest NPV (\$52.2 million)
5. Price of coal \$19/ton growing at 5% per year, extraction cost \$18/ton  
Growing at 3% per year, reserves of \$100 tons, discount rate is 20%, rate of extraction is 10 tons per year

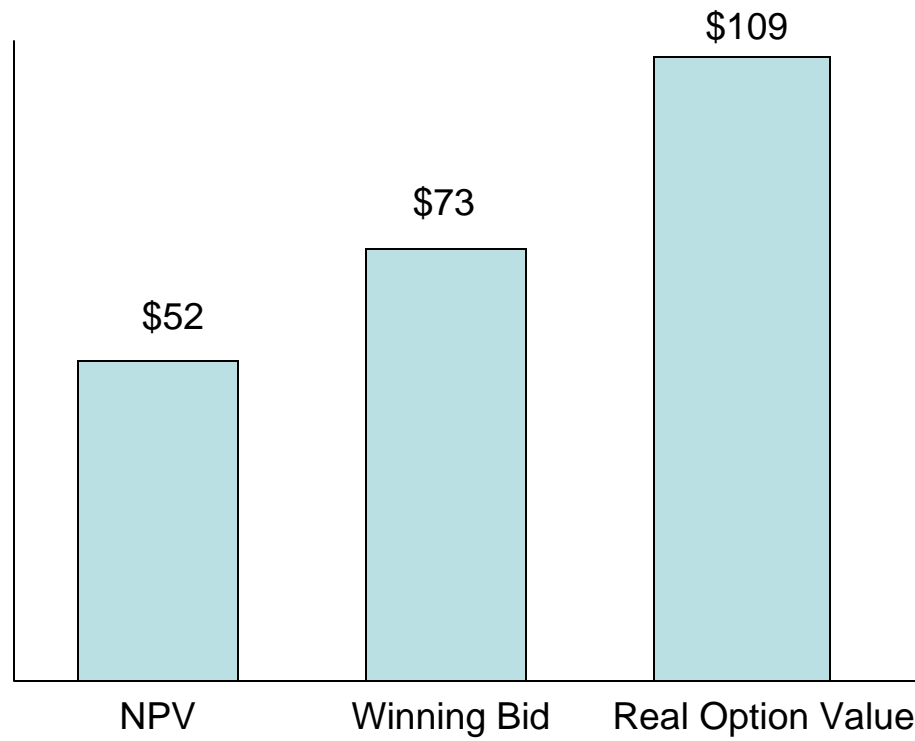
**Table 2 -- NPV Indicates It is Optimal to Invest in Year 2**

Year	Price/ton	Extraction Cost/ ton	Tons mined	Start in Year 1		Start in Year 2			Start in Year 3		
				CF	PV	Tons	CF	PV	Ton	PV	CF
1	\$19.00	\$18.00	10	10.00	8.33	0			0		
2	19.95	18.54	10	14.10	9.79	10	14.10	9.79	0		
3	20.95	19.10	10	18.51	10.71	10	18.51	10.71	10	18.51	10.71
4	21.99	19.67	10	23.26	11.22	10	23.26	11.22	10	23.26	11.22
5	23.09	20.26	10	28.35	11.40	10	28.35	11.40	10	28.35	11.40
6	24.25	20.87	10	33.82	11.33	10	33.82	11.33	10	33.82	11.33
7	25.46	21.49	10	39.69	11.08	10	39.69	11.08	10	39.69	11.08
8	36.23	22.14	10	45.97	10.69	10	45.97	10.69	10	45.97	10.69
9	28.03	22.80	10	52.20	10.21	10	52.20	10.21	10	52.20	10.21
10	29.47	29.49	10	59.89	9.67	10	59.89	9.67	10	59.89	9.67
11	30.95	24.19	0	0	0	10	67.59	9.10	10	67.59	9.10
12	32.50	24.92	0	0	0	0	0	0	10	75.80	8.50
NPV					51.43			52.19			50.90

**Figure 6 -- NPVs of 5 Mutually Exclusive Lease Development Scenarios**



## The Real Option Value was about double

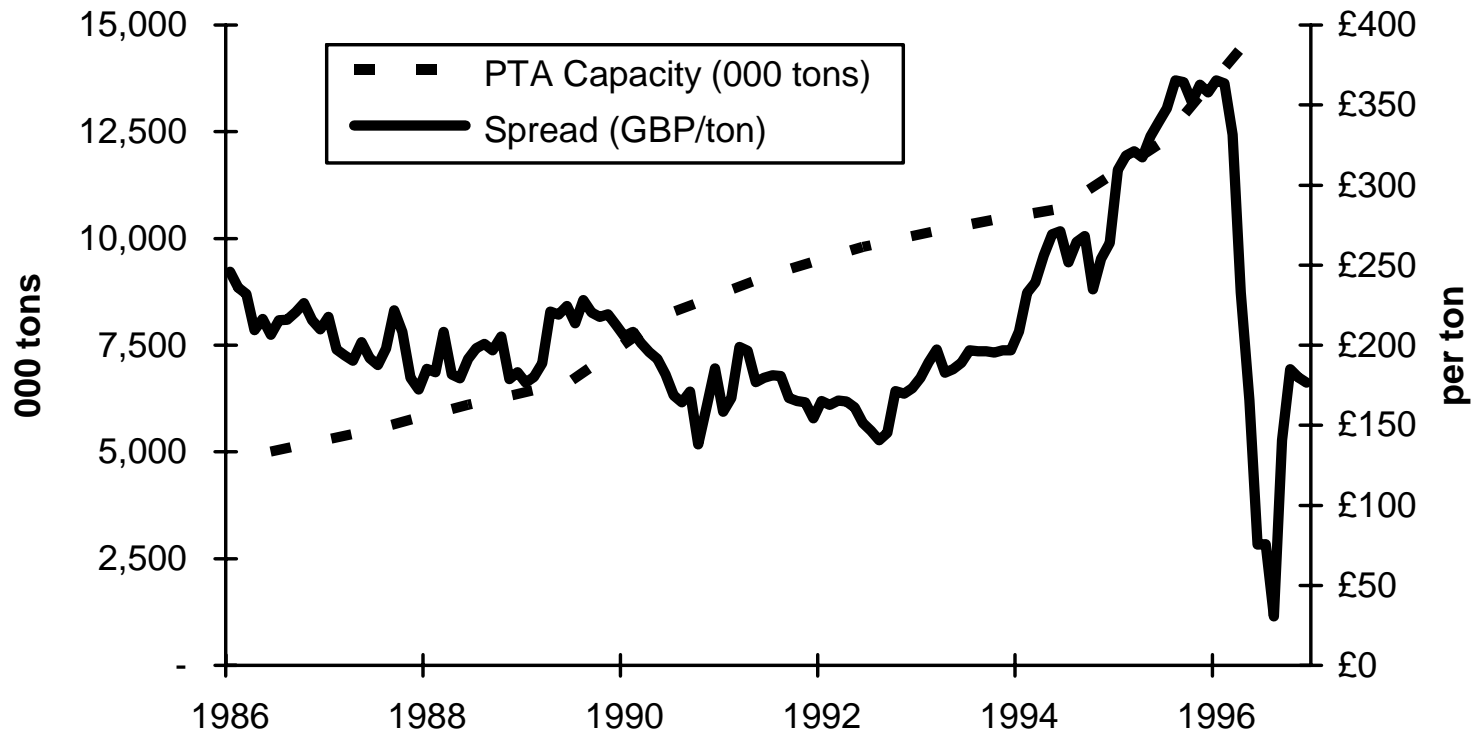


## **Case 2: Cancelable Operating Lease**

- **Manufacturer of jet engines**
- **Intense competition**
- **American put allowing cancellation**
  - **Pre-delivery**
  - **Up to one-year after delivery**
- **Good data on transaction prices**

### Case 3: Multi-phase Investment (sequential compound option)

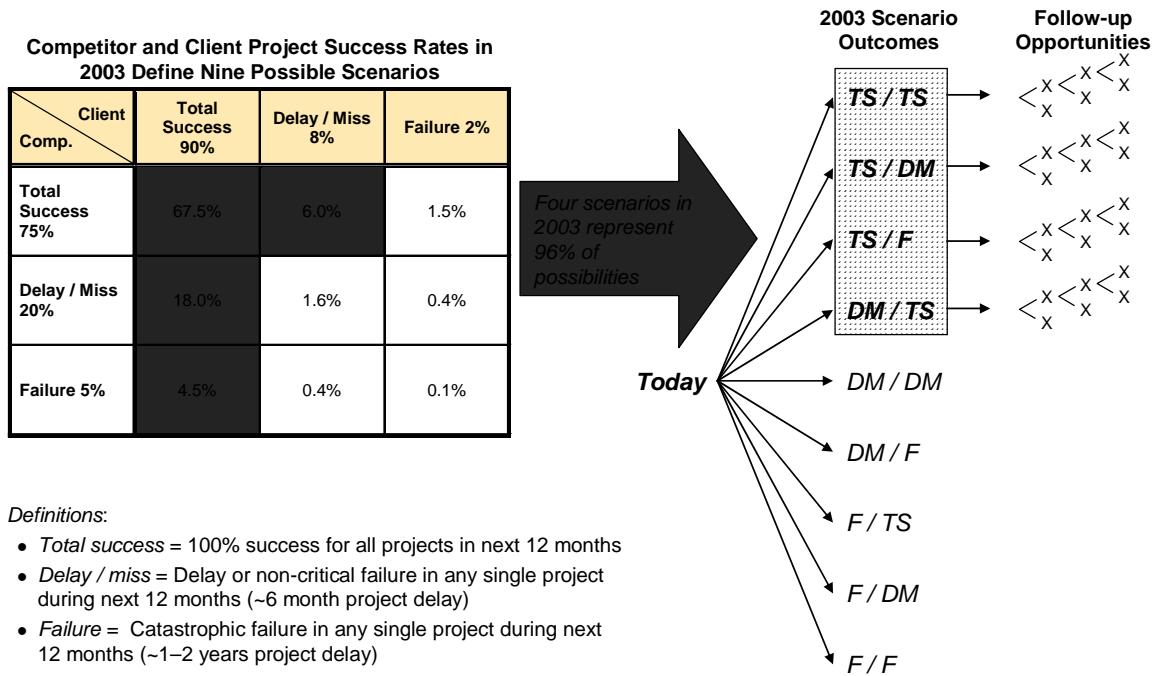
Figure 7 -- Historical PTA Spread Drives Uncertainty



# Case 4: Multi-phase Investment (sequential compound option)

## Figure 8 Scenario Analysis

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## Case 5: Modular Investment

### Figure 10 Capacity Constraints Reduce Expected Supply

Given that demand in the first period can overshoot its expected value by 30 percent or undershoot by 1/1.3, then if a 300 unit plant is built there will be a capacity cap. Similar caps appear in the other periods as well.

**Demand Matrix (units in thousands, probability in parentheses)**

	present	Year 1	Year 2	Year 3
<b>0</b>		390* (0.435)	1014* (0.189)	1977*(0.082)
		231 (0.565)	600 (0.492)	1170*(0.321)
			355 (0.319)	692 (0.417)
				410 (0.180)

\*capacity will be capped

## Table 3 -- NPV Analysis of the Small Plant Alternative

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Standard Net Present Value Calculations, 3 small plants (thousands)

	Present	Year 1	Year 2	Year 3
Expected Output	--	261	522	725
Price per unit	--	2	2	2
Revenue	--	522	1,044	1,450
Variable cost	--	(378)	(757)	(1,051)
Depreciation	--	(100)	(200)	(300)
EBIT	--	44	87	99
Taxes	--	17	35	39
Income	--	26	52	59
Depreciation	--	100	200	300
Salvage value	--	--	--	300
Capital Expense	(300)	(300)	(300)	--
Cash Flow	(300)	(174)	(48)	659
Discount @ 8%	1.000	0.926	0.857	0.794
Present value	(300)	(161)	(41)	523
Net PV	21.3			

Revised NPVs  
1 large \$38.0  
3 small \$21.3

# Figure 11 -- Real Option Valuation of Modular Investments

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## Case 6 – Peak Load Power

- **History of thought**
  - **Margrabe**
  - **Spark spread model**
  - **Add switching costs**
  - **Add changing variance**

# Challenges

- **Game theory and real options**
- **Switching options with inventory**
- **Switching options with multiple modes of operation**
- **Changing variance**
- **Empirical evidence**
- **Better software**